NOTES: SEQUENCES AND SERIES

2. Linear or Arithmetic Sequence: (First difference is a constant)

General Term: $T_n = a + (n-1)d$

a = first term

n = number of terms

d = common difference

Sum of an Aritmetic Series:

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

OR

$$S_n = \frac{n}{2}\left[a + l\right]$$

l = last term

EXAMPLE

Consider the following linear number pattern:

2 ; 7 ; 12 ; 17 ; ....

(a) Determine the nth term and hence the 199th term.

(b) Which term of the number pattern is equal to 497?

Solution:

$a)$ $a = 2$

$d = 5$

$$\begin{aligned} T_n &= a + (n-1)d \\ &= 2 + (n-1)5 \\ &= 2 + 5n - 5 \\ &= 5n - 3 \end{aligned}$$

$$\begin{aligned} T_n &= 5n - 3 \\ &= 5(199) - 3 \\ &= 992 \end{aligned}$$

b) $5n - 3 = 497$

$5n = 500$

$n = 100$

$T_{100} = 497$

EXAMPLE

$Given: x + 4 \; ; \; 2x \;\; x + 8,$

The above is an Arithmetic sequence.

Calculate

The first 3 terms as well as the 12[th] term

Solution:

$$d = T_2 - T_1 = (2x) - (x + 4) = x - 4$$

$$d = T_3 - T_2 = (x + 8) - (2x) = 8 - x$$

$$x - 4 = 8 - x$$
$$2x = 12$$
$$x = 6$$

$10 \; ; \; 12 \; ; \; 14$

$$T_n = a + (n - 1)d$$
$$T_{12} = 10 + (11)2 = 32$$

3. Geometric Sequence: (common ratio)

General Term: $T_n = ar^{n-1}$

a = first term
r = common ratio
n = number of terms

Sum of a Geometric series:

$$S_n = \frac{a(r^n - 1)}{r - 1} \quad ; \quad r > 1$$

$$S_n = \frac{a(1 - r^n)}{1 - r} \quad ; \quad r < 1$$

EXAMPLE

Given: 2 ; 6 ; 18 ; 54

a) Determine the general term.
b) Determine the 10th term.
c) Which term in the sequence will equal 1062882

Solution:

a) $a = 2$

$r = 3$

$$T_n = ar^{n-1}$$
$$= 2(3)^{n-1}$$

$$b)\quad T_n = 2(3)^{10}$$
$$= 118098$$

$$c)\quad 2(3)^{n-1} = 1062882$$
$$(3)^{n-1} = 531441$$
$$(3)^{n-1} = (3)^{12}$$
$$n - 1 = 12$$
$$n = 13$$

EXAMPLE

$7x + 1$; $2x + 2$; $x - 1$ is a Geometric Sequence

a) Determine x
b) Determine the first 3 terms

Solution:

$$a)\quad r = \frac{2x+2}{7x+1} \quad ; \quad r = \frac{x-1}{2x+2}$$
$$\frac{2x+2}{7x+1} = \frac{x-1}{2x+2}$$

$$(2x+2)(2x+2) = (x-1)(7x+1)$$
$$4x^2 + 8x + 4 = 7x^2 - 6x - 1$$
$$3x^2 - 14x - 5 = 0$$
$$(3x+1)(x-5) = 0$$
$$x = \frac{-1}{3} \text{ or } x = 5$$

Sequence 1:

$$\frac{-4}{3};\frac{4}{3};\frac{-4}{3}$$

Sequence 2:

36 ;12; 4

EXAMPLE

3 ; *a* ; *b* are the first three terms of an Arithmetic sequence. If the third term is increased by 3, the three terms form a geometric sequence.
Calculate the values of *a* and *b*.

DO THIS SUM ON YOUR OWN

3.2 Given an arithmetic sequence with $T_1 = 8$ and $T_2 = 11$.

3.2.1 Calculate the value of n if $T_n = 41$. (3)

3.2.2 A new arithmetic sequence P is formed using the term position and the term value of the given arithmetic sequence.
For the new sequence, $P_8 = 1$, $P_{11} = 2$ and so forth.

(a) Write down the value of P_{41}. (1)

(b) Calculate the value of the first term of the new arithmetic sequence. (4)

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QUESTION 2

2.1 Given the geometric series: $\frac{1}{5}+\frac{1}{15}+\frac{1}{45}+...$

2.1.1 Is this a convergent geometric series? Justify your answer with the necessary calculations. (2)

2.1.2 Calculate the sum to infinity of this series. (2)

2.2 An arithmetic and a geometric sequence are combined to form the pattern, which is given by: $P_n = x\,;\frac{1}{3}\,;2x\,;\frac{1}{9}\,;3x\,;\frac{1}{27}\,;...$

2.2.1 Write down the next TWO terms of the pattern. (2)

2.2.2 Determine the general term (T_n) for the odd terms of this pattern. Write down your answer in terms of x. (2)

2.2.3 Calculate the value of P_{26}. (3)

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QUESTION 2/*VRAAG* 2

2.1.1	$\frac{1}{5}+\frac{1}{15}+\frac{1}{45}+\ldots$ $r=\frac{\frac{1}{15}}{\frac{1}{5}}=\frac{1}{3}$ $-1<\frac{1}{3}<1$ $\therefore$ the series is convergent.	✓ $r=\frac{1}{3}$ ✓ answer (any indicator of convergence) (2)
2.1.2	$S_\infty=\frac{a}{1-r}$ $=\frac{\frac{1}{5}}{1-\frac{1}{3}}$ $=\frac{3}{10}$	✓ substitution ✓ answer (2)
2.2.1	$4x\,;\frac{1}{81}$	✓ $4x$ ✓ $\frac{1}{81}$ (2)
2.2.2	$T_n=x+(n-1)x$ $=x+xn-x$ $=xn$ Answer only: Full Marks	✓ substitution ✓ answer (2)
2.2.3	$T_n=ar^{n-1}$ $T_{13}=\frac{1}{3}\left(\frac{1}{3}\right)^{13-1}$ $T_{13}=\left(\frac{1}{3}\right)^{13}$ or $\frac{1}{1\,594\,323}$ or $6{,}27\times10^{-7}$ or 3^{-13}	✓ $n=13$ ✓ $r=\frac{1}{3}$ ✓ answer (3)

3.2.1	$T_n = 8 + (n-1)(3)$ $T_n = 3n + 5$ $41 = 3n + 5$ $36 = 3n$ $n = 12$	✓ $T_n = 3n + 5$ ✓ $T_n = 41$ ✓ answer (3)
3.2.2a	$P_{41} = 12$	✓ answer (1)
3.2.2b	$P_8 = a + 7d = 1$ $P_{11} = a + 10d = 2$ $3d = 1$ $d = \frac{1}{3}$ $a + 7\left(\frac{1}{3}\right) = 1$ $a = -\frac{4}{3}$	✓ $a + 7d = 1$ ✓ $a + 10d = 2$ ✓ value of d ✓ value of a (4)
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